VI
COMPLEX IMPEDANCE

Of course, nothing in the world of AC power is straightforward - we can't just have resistance, we have to have *resistance with imaginary numbers*. An imaginary number is usually written as the letter i (if you're a mathematician) or j (if you're an engineer, because i is taken by current). The imaginary number i or j is equal to $\sqrt{-1}$. The square root of a negative number is a useful concept in math and physics, and especially in AC power. It isn't really "imaginary" at all, since, you know, we're using it for stuff and therefore it exists (in particular, we use it as engineers to represent time delays, or phase shifts). Once upon a time people also thought negative numbers weren't "real."

We'll talk about DC versus AC power in the next chapter, but for now, know that in DC circuits we have resistance and in AC circuits we have a different type of opposition to current flow called complex impedance. This contains both resistance and an imaginary portion called reactance. Reactance also impedes the flow of AC current but it also delays it, hence why we need to use imaginary numbers. Impedance is a combination of resistance (real) and reactance (imaginary). Also in Ohms (Ω), impedance of an AC circuit (like the power grid) changes with the frequency of the AC power and comes from two types of impedances that can be thought of as energy storage devices: *inductors* and *capacitors*. Examples are shown in Fig. ix).

Capacitors. Capacitors are components that store energy in *electric fields*. In the image below, if you rip open the cylinders you will see sheets of conductive material wrapped around insulating material. The capacitor "holds electric charge" by maintaining a voltage drop across the conductive plates (sheets).

Inductors. Inductors are components that store energy in *magnetic fields*. They are essentially coils of wire wrapped around an insulator. Unlike the capacitor which can help maintain a constant voltage drop, inductors help regulate steady current flow through a circuit.

FIG. ix Capacitors (left) ranging from baby (top) to momma (bottom); inductors (right).

Capacitors and inductors are used in both DC and AC circuits, but we'll focus on AC here. Let's first talk about some practical uses for these components.

Capacitors in Air Conditioner Condensers. If you have a central air conditioning (A/C) system, you probably have noticed the outdoor unit, usually referred to as the condensing unit. Figure x shows the insides of a 26-year old condensing unit that was struggle-bussing to cool last summer and thus the owner decided to replace it. The zoomed-in portion shows two capacitors: the "start" capacitor, on the right, and the "run" capacitor, on the left.

The compressor motor inside this unit (and many AC motors in general) need an initial "jolt" of energy to get moving that can't necessarily come from your home's electrical system. Once they have overcome this activation energy, it's easier for them to continue moving. The start capacitor provides this initial high voltage, and the run capacitor makes sure that a consistent voltage is supplied to the motor as it operates, helping it continue to run smoothly.

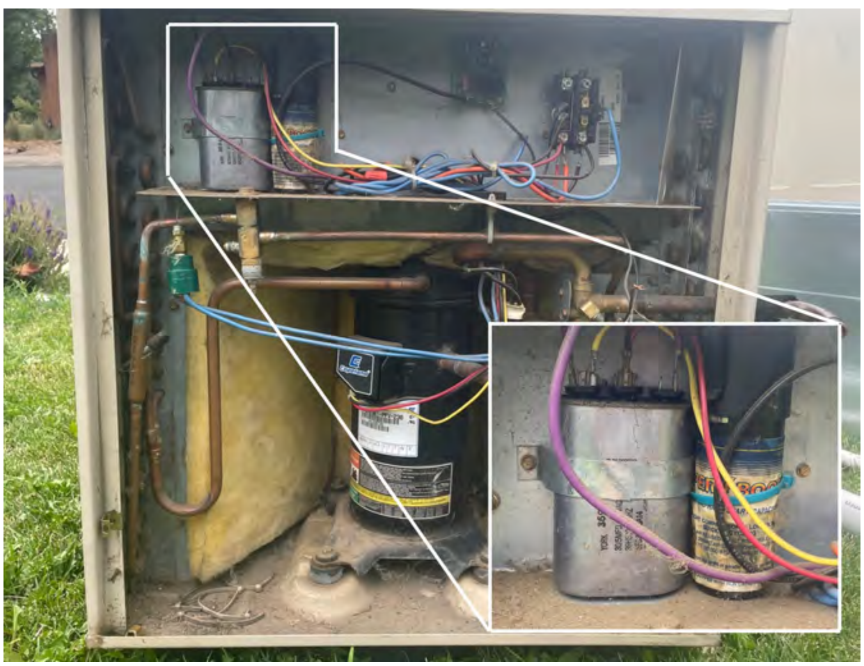

FIG. x A/C condenser units use two capacitors: a start (right) and a run (left) capacitor.

Induction motors. Speaking of motors, the compressor motor mentioned above that needs a capacitor to start also relies on the concept of induction to run. In fact, most motors are *induction motors*, which means that they rely on inductors to spin. See Fig. xi, with a small fan motor on the left and larger dishwasher motor on the right.

FIG. xi An induction motor for a small fan (left) and an induction motor for a dishwasher (right), with capacitor indicated by the arrow.

As an alternating current passes through the coil of wire, it generates a time-varying magnetic field. In the left image, electric current is passed through the inductor (the coil of copper wires). This part stays stationary and does not physically move, and is thus called the *stator* of the motor. A magnetic field is then generated, causing the *rotor*, or the rotational part of the motor (in this case, connected to the fan), to spin as it attracts and repels the magnetic field in the stator.

Complex Resistance. There's a "native" unit for capacitance and inductance, though - capacitors are rated in *Farads* and inductors are rated in *Henrys*. By using imaginary numbers and other mathematical manipulations, however, we can convert these units into Ohms. Doing so allows us to treat these components like resistors, and calculate things like (RMS) voltage and current in a circuit without resulting to things like differential equations. Table 2 shows these conversions, where j is $\sqrt{-1}$ and f is the frequency, in Hertz (Hz), of the AC voltage and current through the component. R is the value of a resistor (in Ohms), L is the value of an inductor (in Henrys), and C is the value of a capacitor (in Farads).

Table 2 Impedance (Ω) of different components

Component	**Impedance (Ω)**
Resistor	R
Inductor	$2\pi jL$
Capacitor	$\frac{1}{2\pi jC}$

Total impedance is the sum of the overall resistance plus the imaginary number times the overall reactance. More information and a full derivation of complex impedance can be found in a circuits textbook.

The benefit of converting capacitors and inductors into Ohms is that now we can use the famous *Ohm's Law* to establish relationships between these components, voltage, and current.

Ohm's Law. For circuits without any inductors or capacitors, Ohm's law states the following:

$$V_i - V_j = I_{ij} \cdot R_{ij} \tag{5}$$

Which says that the voltage drop between points i and j in a circuit is equal to the current flowing between i and j and the resistance to current flow between these two points. You may have seen this written in a shorter form: $V = IR$. I prefer to write it in the above way because it reminds us that voltage must be measured between two points.
For circuits with inductors and capacitors, which is like, most systems you'll encounter in practice that use AC power, Ohm's law is $V = IZ$, or

$$V_i - V_j = I_{ij} \cdot Z_{ij} \tag{6}$$

Now we have complex impedance instead of just resistance (voltage and current can also be complex numbers, FYI!). The complex part is sometimes annoying to deal with, so if the amount of inductance and capacitance in a system is small, we sometimes just ignore it and use $V = IR$. This is done when residential building electrical systems are designed, for example.

It's worth noting that Ohm's law actually doesn't hold in every single circuit, and it's only an approximation of what happens in practice. Things that aren't metal conductors, have constant temperatures, or constant impedance (like transistors and diodes, which are in most electronic devices you own) don't obey Ohm's law. In general, Ohm's law represents a linear relationship between current and voltage (if you plotted current vs. voltage it would be a line with a slope equal to the impedance). These other devices introduce nonlinearities.